June 2017:

QUESTION 3

Given the quadratic sequence: 0; 17; 32; ...

3.1   Determine an expression for the general term, T_n, of the quadratic sequence.   (4)

3.2   Which terms in the quadratic sequence have a value of 56?   (3)

3.3   Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$.   (4)

     [11]

June 2017:

Q3.1

First differences: 17; 15
Second difference: −2

$$T_n = an^2 + bn + c$$

$$a = \frac{\text{second difference}}{2} = -1$$

$$3a + b = 17$$
$$3(-1) + b = 17$$
$$b = 20$$
$$a + b + c = 0$$
$$-1 + 20 + c = 0$$
$$c = -19$$
$$T_n = -n^2 + 20n - 19$$

Q3.2

$$56 = -n^2 + 20n - 19$$
$$n^2 - 20n + 75 = 0$$
$$(n - 15)(n - 5) = 0$$
$$n = 5 \ \text{ or } \ n = 15$$

Q3.3

$$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$$

$$= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T_{15}$$

$$= (T_5 - T_{15}) + (T_6 - T_{14}) + ... + (T_9 - T_{11}) + T_{10}$$

$$= T_{10}$$

because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ ...

$$T_{10} = -(10)^2 + 20(10) - 19$$
$$= 81$$

Nov 2019

QUESTION 3

3.1 Without using a calculator, determine the value of: $\displaystyle\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1}$

Nov 2019

Q3.1

$$\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1}$$

$$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{8}\right) - \left(\frac{1}{2} + \frac{1}{3} + ... + \frac{1}{8} + \frac{1}{9}\right)$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9}$$

2.2 Given a geometric sequence: $36 ; -18 ; 9 ; \ldots$

    2.2.1 Determine the value of $\dot{r}$, the common ratio.

    2.2.2 Calculate n if $T_n = \dfrac{9}{4\,096}$

2.2.3 Calculate S_∞

2.2.4 Calculate the value of $\dfrac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$

MAY /JUNE 2019

Q2.2.1	Q2.2.3
$r = \dfrac{-18}{36} = -\dfrac{1}{2}$	$S_\infty = \dfrac{a}{1-r}$
Q2.2.2	$= \dfrac{36}{1 - \left(-\dfrac{1}{2}\right)}$
$T_n = 36\left(-\dfrac{1}{2}\right)^{n-1}$	$= 24$
$\dfrac{9}{4096} = 36\left(-\dfrac{1}{2}\right)^{n-1}$	**Q2.2.4**
$\dfrac{1}{16384} = \left(-\dfrac{1}{2}\right)^{n-1}$	$\dfrac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$
$\left(-\dfrac{1}{2}\right)^{14} = \left(-\dfrac{1}{2}\right)^{n-1}$	$= \dfrac{a + ar^2 + ar^4 + \ldots + ar^{498}}{ar + ar^3 + ar^5 + \ldots + ar^{499}}$
$14 = n-1$	$= \dfrac{a + ar^2 + ar^4 + \ldots + ar^{498}}{r\left(a + ar^2 + ar^4 + \ldots + ar^{498}\right)}$
$n = 15$	$= \dfrac{1}{r}$

EXAMPLE

The constant second difference of the quadratic number pattern:
$4 ; x ; 8 ; y ; 20 ; \dots$ is 2.
(a)   Determine the value of x and y.
(b)   Determine which term equals 125.

DO THE SOLUTION

EXAMPLE

$3 ; a ; b$ are the first three terms of an Arithmetic sequence. If the third term is increased by 3, the three terms form a geometric sequence.
Calculate the values of a and b.

DO THIS SUM ON YOUR OWN

June 2018

QUESTION 2

2.1   Given the quadratic pattern: $5 ; 10 ; 17 ; 26 ; \dots$

      2.1.1   Write down the next TWO terms of the pattern. (2)

      2.1.2   Determine the formula for the nth term of the pattern. (4)

      2.1.3   Which term of the pattern will have a value of 1 765? (4)

2.2   The first 24 terms of an arithmetic series are: $35 + 42 + 49 + \dots + 196$.

    Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)

    [15]

Arithmetic series and sequences

June 2018

Q2.1.1

37 ; 50

Q2.1.2

$$a = \frac{\text{second difference}}{2} = \frac{2}{2} = 1$$

$$3a + b = 5$$
$$3 + b = 5$$
$$b = 2$$
$$a + b + c = 5$$
$$1 + 2 + c = 5$$
$$c = 2$$
$$T_n = an^2 + bn + c$$
$$= n^2 + 2n + 2$$

Q2.1.3

$$n^2 + 2n + 2 = 1765$$
$$n^2 + 2n - 1763 = 0$$
$$(n + 43)(n - 41) = 0$$
$$n = -43 \text{ or } n = 41$$
$$\text{N/A}$$

Q2.2

Sum of all multiples of 7 from 35 to 196:

$$a = 35; \quad d = 7$$
$$S_n = \frac{n}{2}[a + \ell]$$
$$= \frac{24}{2}[35 + 196]$$
$$= 12[231]$$
$$= 2772$$

Sum of all the natural numbers from 35 to 196:

$$a = 35; \quad d = 1; \quad n = 162$$
$$S_n = \frac{n}{2}[a + \ell]$$
$$= \frac{162}{2}[35 + 196]$$
$$= 81[231]$$
$$= 18\,711$$

Sum of numbers not divisible by 7/
Som van getalle nie deelbaar deur 7
$$= 18\,711 - 2772$$
$$= 15\,939$$

QUESTION 3

A quadratic sequence has the following properties:

- The second difference is 10.
- The first two terms are equal, i.e. $T_1 = T_2$.
- $T_1 + T_2 + T_3 = 28$

3.1   Show that the general term of the sequence is $T_n = 5n^2 - 15n + 16$. (6)

3.2   Is 216 a term in this sequence? Justify your answer with the necessary calculations. (3)

[9]

QUESTION 3/VRAAG 3

3.1		
	$x \; ; \; x \; ; T_3 \; ; \dots$	
	$0 \qquad T_3 - x$	
	10	
	$2a = 10 \qquad 3a + b = 0$	$\checkmark 2a = 10$
	$\quad a = 5 \qquad\qquad b = -15$	$\checkmark 3a + b = 0$
	$T_3 - x - 0 = 10$	
	$\therefore T_3 = x + 10$	$\checkmark T_3 = x + 10$
	$2x + T_3 = 28$	$\checkmark 2x + T_3 = 28$
	$2x + x + 10 = 28$	
	$3x = 18$	
	$x = 6$	$\checkmark x = 6$
	$a + b + c = 6$	
	$5 - 15 + c = 6$	$\checkmark 5 - 15 + c = 6$
	$\qquad c = 16$	
		(6)
	$\therefore T_n = 5n^2 - 15n + 16$	
	OR/OF	**OR/OF**
	$2a = 10$	$\checkmark 2a = 10$
	$\therefore a = 5$	
	$T_1 = a + b + c \quad T_2 = 4a + 2b + c \quad T_3 = 9a + 3b + c$	
	$\quad = 5 + b + c \qquad = 20 + 2b + c \qquad = 45 + 3b + c$	
	$5 + b + c = 20 + 2b + c$	$\checkmark 5 + b + c = 20 + 2b + c$
	$\qquad b = -15$	
	$T_1 = -10 + c \qquad T_2 = -10 + c \qquad T_3 = c$	$\checkmark T_1 = -10 + c$
		$\checkmark T_2 = -10 + c$
	$T_1 + T_2 + T_3 = -10 + c - 10 + c + c$	
	$\quad 28 = 3c - 20$	$\checkmark 28 = 3c - 20$
	$\qquad c = 16$	$\checkmark c = 16$ $\qquad$ (6)